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NZQA | Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 3 Mathematics and Statistics (Statistics) 2024

91586 Apply probability distributions in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) A person's belly button can be described as an "innie" (navel indented) or an "outie" (navel protruded), as pictured below. Belly-button shape is a cosmetic characteristic and not a significant indicator of health, so data about belly buttons is not routinely collected. It is therefore difficult to make a reliable estimate of the proportion of outie belly buttons in the population.

https://www.pampers.com/en-us/baby/newborn/article/outie-belly-button

Data on belly-button type was collected in the world's first belly button survey. From this data, it was found that 96% of respondents had an innie belly button.

(i) Using an appropriate probability distribution model, calculate an estimate for the probability that, in a class of 30 students, no students have an outie belly button.

State the probability distribution model and the parameters used as part of your answer.

(ii) Give TWO reasons why the use of the probability distribution model selected in part (i) is appropriate.

Reason one:

Reason two:

(iii) Belly button fluff is the accumulation of fibres, lint, or debris that collect in a person's belly button. A belly button survey found that 66% of all the respondents had fluff in their belly buttons.

Using an appropriate probability distribution model, calculate an estimate for the probability that one student in a class of 30 has an outie belly button, and it contains belly button fluff.

You should state the probability distribution model and the parameters used as part of your answer.

(b) Very little research on the typical size of belly buttons has been undertaken. It is hypothesised that the diameter of adult belly buttons (the distance across at the widest point) could be modelled by a normal distribution with mean of 2.05 cm and standard deviation of 0.9 cm.

(i) Calculate an estimate for the probability that a randomly selected adult belly button has a diameter of at least 1.75 cm, given that its diameter is between 1.5 cm and 2.0 cm.

(ii) Comment on the suitability of using the parameters given above for modelling adult belly button width.

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The assessment continues on the following page.

QUESTION TWO

Fluff collects in the belly buttons (navels) of some people. The Nature of Navel Fluff study (Steinhauser) was published in 2009. The author of the study collected data about his own belly button fluff. Each day, for 502 days, he had a morning shower where he washed his belly button. At the end of each day, he removed and weighed the belly button fluff that had accumulated during the day.

(a) The weight distribution of the amount of belly button lint (fluff) he collected each day is shown in Figure 2(a) below.

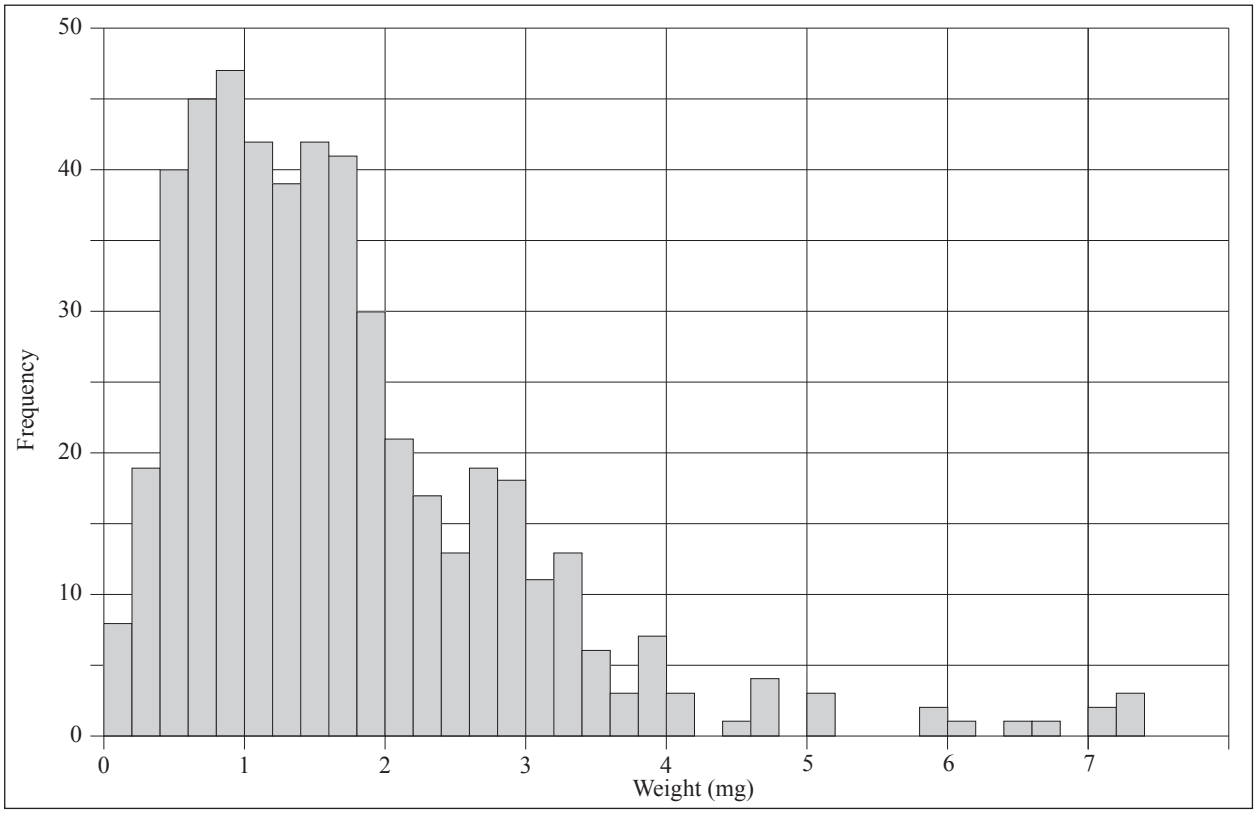

Figure 2(a): Weight distribution of 502 pieces of daily belly button fluff

It is proposed to model daily belly button fluff weight using a triangular distribution with parameters:

minimum = 0 mg, maximum = 7.4 mg, and mode = 1.0 mg

(i) Sketch this model on Figure 2(a).

(ii) Using the given parameters, estimate the probability that on any given day the belly button fluff weight is less than 1.5 mg.

(iii) Suggest alternative parameters for a triangular distribution that will give a better fit to this data and sketch your new model onto Figure 2(b) below.

As part of your answer, justify your choice of these alternative parameters.

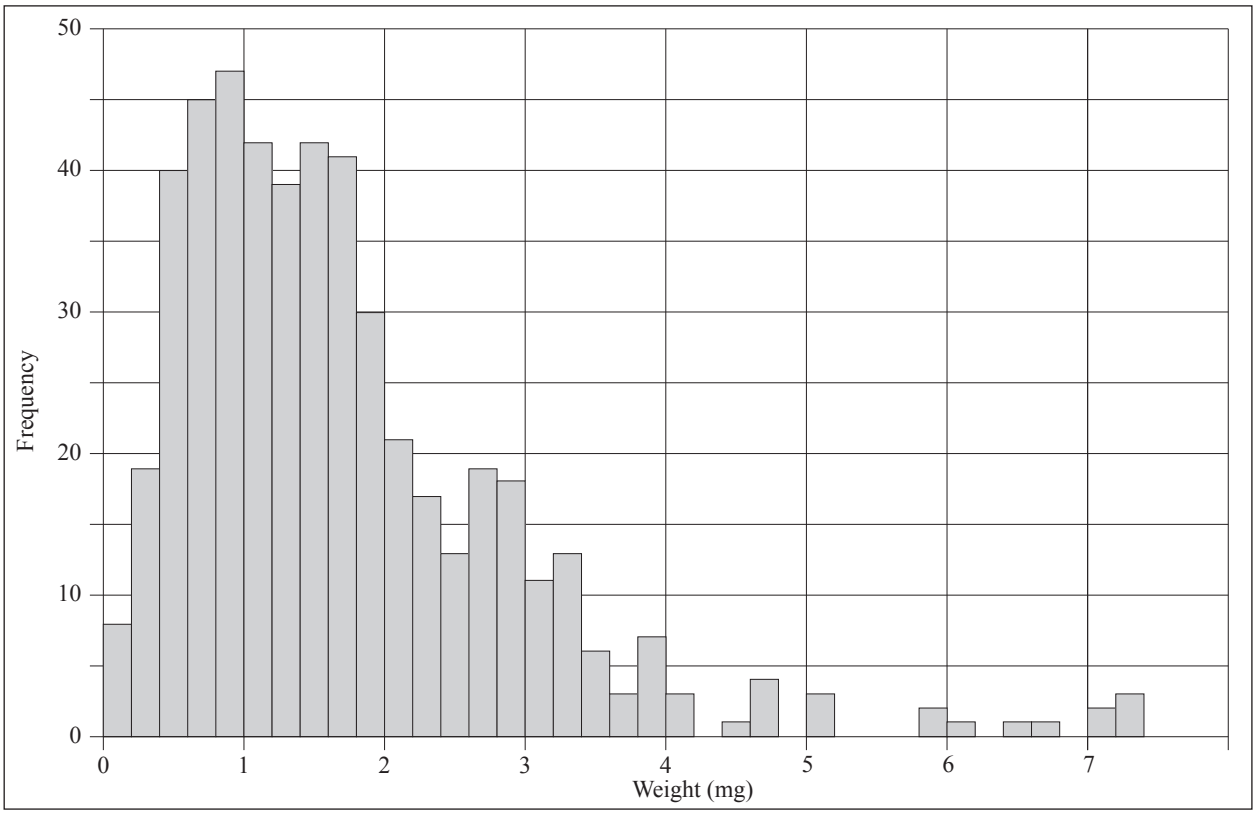

Figure 2(b): Weight distribution of 502 pieces of daily belly button fluff

(b) The table below shows the probability distribution of the random variable, T, the number of towels used per person for residents from one city over one day.

t	0	1	2	3	4	5	6
P(T = t)	0.129	0.271	0.264	0.185	0.095	0.039	0.017

(i) How many towels, on average, were used by a resident from this city over the one day?

(ii) Let the random variable, C, be the number of sets of clothing worn for residents from this city on this one day. The maximum number of clothing changes by residents of this city on this day is 3.

The random variable C has SD(C) = 0.5.

Show that C has a lower standard deviation than T, and give one reason why this might be the case.

(iii) SD(T + C) = 1.754

Are T and C independent?

Comment on what this suggests about the number of towels used and number of clothing changes for residents of this city.

Support your answer with statistical statement(s) and calculation(s).

QUESTION THREE

A study published in 2012, based on data collected from 138 people participating in a nationwide American citizen science project in 2011, collected information about belly button habits. The distribution of washing frequency (defined as the number of belly button washes per 7-day week), is shown in Figure 3(a).

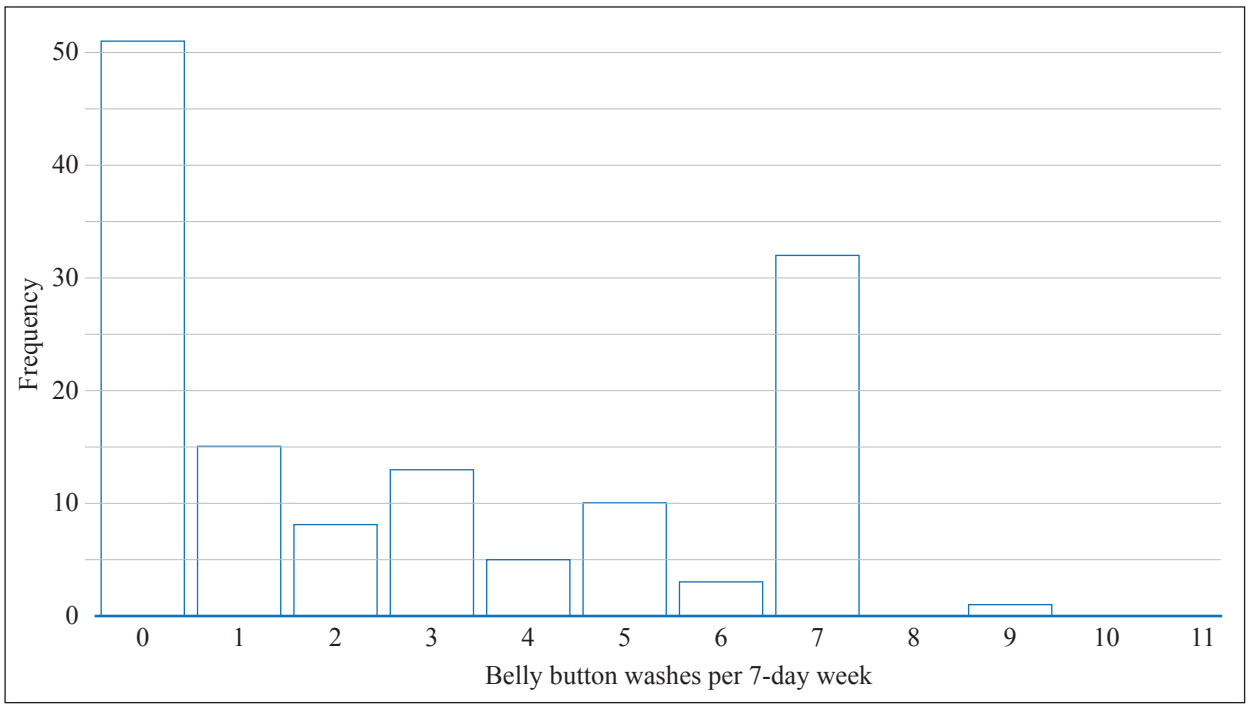

Figure 3(a): The number of reported times survey respondents washed their belly buttons per week

(a) A media release reporting on the study's results had the following headline:
"SHOCKING – less than a quarter of US citizens wash their belly button daily".

Comment on whether this headline is correct.
Support your answer with statistical calculations and reasoning.

(b) Suppose the number of belly button washes per week for the general population is modelled by a Poisson distribution with $\lambda = 2.8$.

(i) Use this Poisson model to calculate an estimate for the probability that a randomly selected person from the general population washes their belly button at least once in a day.

(ii) To apply the distribution used in part (b)(i), at least one assumption must be made.

Identify ONE assumption made that may be invalid and discuss why this is the case.

Question Three continues on the next page.

The sample data shown in Figure 3(a) is run through a simulation model 1000 times, assuming that a Poisson distribution with $\lambda = 2.8$ is used to model the number of belly button washes per 7-day week for the general population. Figure 3(b) shows the results of the simulation model and the original observed data (blue dots).

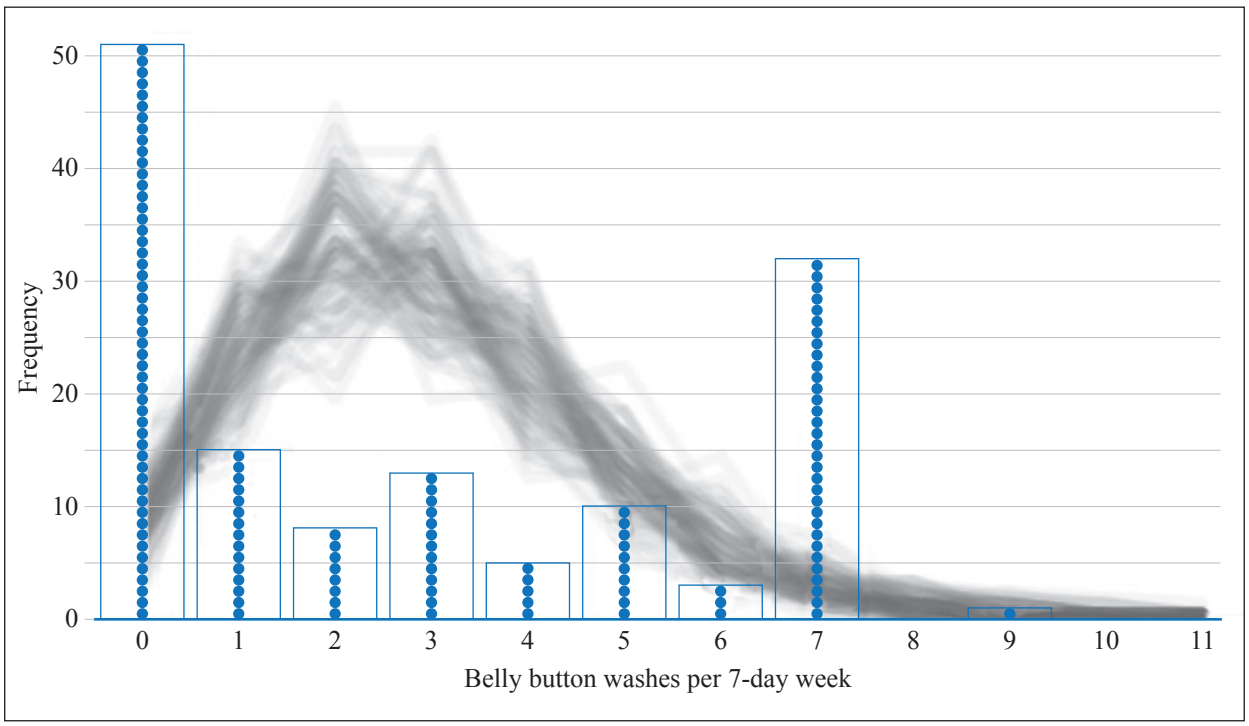

Figure 3(b): Results of the simulation model and original observed data

(iii) Explain what the grey band is showing in Figure 3(b).

(iv) Based on the results of the simulation model and the original observed data (Figure 3(b)), discuss whether the Poisson distribution model (presented opposite) appears to be appropriate for modelling the number of belly button washes per 7-day week.

Support your answer with statistical reasoning.

(v) Propose and justify the use of an alternative model that could be appropriate for modelling the number of belly button washes per 7-day week.

You should state the probability distribution model and the parameters used as part of your answer.

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